Mathematics: A Prodigy of Nature

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Abstract: *This paper tries to establish the deep connection of Mathematics to Nature. The discipline of mathematics developed alongside human civilisation so as to facilitate mankind to coexist with nature. It not only helped to explain the intricate laws of nature but also imbibed the essence of these laws to evolve in a logical and rational manner. The focus of this article will be on the characteristics that are inherent in the natural phenomena that have been truly imbibed and explained by Mathematics so much so that the subject can be called the true prodigy of nature.*

Keywords: Cantor, Infinity, Set Theory, Fibonacci, Fractals, Mobius Band, Paradoxes, Pi, One-to-One Correspondence, Chaos Theory, Euclid's Fifth Postulate

1. Introduction

Right from the advent of civilization, man had been in search of language through which he could understand the different facets of nature that influenced his life in different ways. Ever since the discovery of fire and invention of the wheel, laws of nature have governed man's life. Nature and its laws are inseparable and they work in harmony with each other (Bigelow et al., 1992; Maturana, 2000). If we can acquire a thorough understanding of the natural phenomena, it will enable us to overcome the tendencies to break the laws of nature and as a result generate misery for ourselves. Nature, not only governs our physical existence but plays a much more intense and significant role at the level of the mind where we form our habit patterns, attitudes, perspectives and a sense of intuition. Therefore, it becomes even more imperative to comprehend the laws of nature and their implications (Dretske, 1977; Marwaha, 2016).

From the beginning of civilization, man has observed that nature is so disciplined and logical in behaviour that even the seemingly chaotic patterns contain in them the elements that explain both physical and metaphysical truths of our existence (Schaeffer, 2017). Mathematics as a subject, is a true reflection of nature. In fact, it can be called the 'language of nature' because the journey of mathematics started with the discovery of numbers for which nature has been the source of inspiration.

"Nature's great book is written in mathematical language" – Galileo Galilei (Peat, 1990).

In order to quantify the physical world we live in, counting numbers were devised. About 7000 years ago, the Sumerian race that resided in Mesopotamia developed the first form of writing and recording numbers (Struik, 1987; Kramer, 1995). As man grew crops and became a potter after inventing the wheel, trade flourished and so did mathematics (Sarton, 1936). Calculation became an integral part of man's life and consequently the scope of expanding the horizons of mathematics also increased. Ancient Egyptians devised a number system using the base as 10 and formed symbols to depict the numbers 1,10, 100, 1000, 100,000 and 1,000,000. The first indications of mathematical thinking were when the plethora of stones all around were recognized as potential tools to do counting of finite quantities (Law, 2012). This led to the mathematical concept of one-to-one correspondence. But the mind is always in the quest of new areas of knowledge. So, reasonably enough, man was not satisfied with finite numbers. Again, man looked towards nature for inspiration because nature conceals the seed of infinity in finiteness. A single seed of grain has the potential to generate an endless amount of grain. Scientists, in the quest for the ultimate truth of matter discovered that it could be broken into many small sub-atomic particles. But the search had been for the smallest indivisible particle that forms the basic building block of matter. The attempts made by several scientists finally resulted in the conclusion that such a particle is nothing solid or concrete but a manifestation of energy. This was the discovery of science regarding the ultimate reality of matters. When this transition from boundedness to unboundedness, from countable to uncountable, from measurable to immeasurable is happening in nature, then could it be that mathematics would not imbibe this quality and transcend finiteness to discover infinity (Marwaha, 2016).

Just as nature is governed by its own laws, mathematics also has its structure resting firmly on the foundation of principles and axioms (Dossey, 1992). Interestingly, as nature is endowed with laws like those of gravity, the basic axioms of Mathematics are also inherently present in it. The best example to illustrate this fact is that of numbers. It was only needed to denote a single object by number 1 and the rest of the numbers inevitably followed. It is also true that to list all the counting numbers is an impossible task. Therefore, the axiom was laid down "Given any counting number, however large, there exists a counting number greater than it". This is a principle which has to be accepted without proof. Its veracity is irrefutable like the law of gravity which is the basic functioning principle of nature (Schoenfield, 1967; Marwaha, 2016).

The rigor of nature cannot be challenged and likewise the precision of mathematics is unquestionable. Nature conforms to truth because it develops according to its own laws which are irreversible. Moreover, these laws cannot be conceived. Those who have the curiosity of knowing the truth can perceive them and try to comprehend them. In this aspect, Mathematics is also greatly similar to nature. Mathematics cannot be claimed by anyone to have been invented. It was discovered in the natural course of evolution as mankind felt an urgent need to provide a language for the silent grandeur of nature. The claim of mathematics to have existed as a rationalistic expression of the behavioural pattern of nature will be amply illustrated in the rest of the article (Hunt, 2006; Marwaha, 2016).

2. One-to-One Correspondence: The Birth of Numbers

Once again, let us go back to the era when civilization was still in its cradle and man had just begun to lead a settled community life, growing crops or rearing cattle. He had no notion of measuring quantities whether discrete or non-discrete, either through numbers or through any other units of measurement. But his need was imminent and unavoidable (Sarton, 1936; Gullberg, 1997).

Imagine a shepherd or a farmer who sends his cattle to the open fields for grazing and collects them again in the evening. He wants to be sure that he has not lost any cattle during the day but he has no means of counting them as numbers were not known at that time. So what does he do? He devises an ingenious method for keeping track of the number of cattle. Every morning as he brings out a cow from the barn, he keeps a pebble (or a stone) for that cow. Again, for the next one, he puts another pebble. Thus putting one pebble for each cow, he has a pile containing as many pebbles as the cows. In the evening, when the cows return home, before sending them into the barn, for each cow, he would remove exactly one pebble. If any pebbles are remaining, that would mean those many cows got lost. And if there were extra cows with no pebbles against them, that would mean that some other farmer's cow had strayed into his herd (Marwaha, 2016).

It is evident that the mathematical idea inherent in this simple yet elegant method adopted by the shepherds was one-to-one correspondence. Soon it was observed that to apply the idea of one-to-one correspondence, the nature of elements was immaterial. The cows could be replaced by oranges, apples, chairs etc. as long as the collection of these objects was of the same size. These collections of objects came to be denoted as 'sets'. The pressing need to count or measure led to the idea of assigning a symbol to every set having a fixed size and that is how the birth of numbers took place. A set containing one object was denoted by a particular symbol; the set of two objects by another symbol and so on. These symbols were different in different civilizations. The natural numbers denoted by the numbers 1, 2, 3... are a result of this line of thought, which are just symbols to denote a set containing those many objects. Thus, a natural number n stands for a set containing n elements. Also observe the beautiful way in which these numbers have been defined. Starting from the number 1, the successive number 2 is obtained by adding exactly 1 and similarly 2 plus 1 gives 3 and so on. This means that a natural n number can be written as

$n = 1 + 1 + 1 + \ldots + 1,$

where 1 is added n number of times. This clearly represents adding the numbers in the most natural manner. For instance, 2 added to 3 means,

$2 + 3 = (1 + 1) + (1 + 1 + 1) = 1 + 1 + 1 + 1 + 1$
= addition of 1 five times, and so must be equal to 5.

Next we see how nature shows the path from finiteness to infiniteness.

3. Transcending Finiteness

Every natural number n is a finite quantity in itself because it symbolizes a collection of n number of objects. The next natural number can be obtained by adding the number 1 to n and is denoted by $n + 1$. Moreover, it is greater than the preceding natural number n as it symbolizes a set containing one element more that the set containing n objects. This process shows that given any natural number n, there will always exist a natural number greater than it. The evidence that nature could have been the source of inspiration for this simple way of counting is the stumps of trees. If we look at the cross section of the trunk of a tree, we will find circles. Each year, a new circle is formed on the stump of a tree and the total number of circles indicate the age of the tree. Nature likes to add one circle to indicate the growth of the tree in years (Marwaha, 2016).

Let us see how mathematics transcends the gross finite reality to discover truth in the subtle infinite depths. We begin with number 1 and observe that it can be broken down into as many pieces as we like. For example: -

$$1 = \frac{1}{2} + \frac{1}{2}$$

$$1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$1 = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \quad \text{and so on.}$$

This is only one illustration. There can be many ways in which the number 1 can be expressed as a sum of a finite number of fractions. Clearly, this can go on indefinitely but at each stage, the partition is into smaller but finitely many parts, that is, parts that can be counted. The claim of mathematics to be close to nature would have been false if it did not know how to transcend the barriers of finiteness. Thus, mathematics discovered infinity and it was found that finite numbers could be broken into infinitely many pieces. The number 1 can be expressed as the infinite sums.

$$1 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\cdots$$

$$1 = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots\cdots$$

$$1 = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots\cdots \quad \text{and so on.}$$

This also demonstrates that any counting number can be written as an infinite sum of smaller finite numbers. Although the barriers of finiteness had been broken, but the infinity discovered was countable infinity i.e., the infinity of natural numbers or commonly known as counting numbers. In the first set of equations given above, one can see the beginning and the end of the finite sum but in the second set of equations, there is no end to the infinite sums. Infinite sums lend a sense of continuity, which matches well with the analogous phenomena present in the natural way of things (Marwaha, 2016).

Many philosophers, mathematicians and scientists have battled with the question of finiteness and infiniteness of the universe and each has come to his own derivations based on his perspective of the problem. Archimedes (287-212 BC) of Syracuse also tackled this intriguing problem to count the number of sand grains in the universe. Using some simple properties of geometry and making fine assumptions regarding the size of the earth, sun and moon, Archimedes arrived at the same conclusion that the sand particles contained in the entire universe are indeed countable and finite. Archimedes' reason for solving this problem was twofold – one, to satisfy the natural curiosity of his King Gelon about the size of the universe but the other was more pedantic. By undergoing this exercise, Archimedes wanted to develop a number system through which very large numbers could be represented.

Archimedes' argument to prove his assertion was that if the grains of sand are infinite, then the universe must be infinite and so every part of the universe must be filled with sand. Since this is not the case, sand particles are finite in number. This brings us to the question, "Is the world that we live in, finite or infinite?"

Before we begin to contemplate on this subject, the definition of the word 'finite' must be made clear. In layman's language, the word 'finite' means 'something that is measurable in terms of size or countable in terms of quantity'. In a more philosophical mode, it would mean that which is easily comprehensible by the mind.

Although it seems quite improbable to call the ever growing and ever expanding universe around us finite, but we human beings have already endeavoured to capture the essence of vastness and make it more viable and workable for ourselves. Infinity may have appeared to be elusive but this has not deterred us from attempting to attain it or at least to define it in more feasible ways. But then, how does one ever begin the process of assimilation? Mathematics seems to have found its own solution to this seemingly insurmountable problem and in doing so, it is again following the path shown by nature.

Consider the example of the number of leaves growing on, say, a banyan tree. If someone undertakes the formidable task of counting these without setting any deadlines, would he be able to do so? A very persistent person might claim to be able to do it but probably by the time he has finished counting, the spring season would be bringing out a fresh set of leaves. Let us grant that the person has made it his lifetime errand to count the leaves on a given tree during the life history of that tree. His aim is to prove that the process of counting would eventually come to an end with the death of the tree and so the answer arrives at is a finite one. Can the same conclusion be reached if all the leaves growing on the various plants and trees in the entire planet earth have to be counted? The idea of finiteness here takes a severe beating. What was workable for a single tree (that too a purely hypothetical situation, just to illustrate a point) is certainly not possible in the latter case, not because of the lack of resources but due to the undeniable fact that in the universe, the process of destruction and birth is taking place incessantly.

Although this scenario may be called finite, man needed to change his perspective so that he could gauge to some extent the magnitude of the situation. Therefore, instead of looking at leaves as separate entities, he preferred to shift his focus to the tree as a collection of leaves. The task of counting the trees is far less daunting, than counting the number of leaves. Further, in order to make our planet earth look even more quantifiable, the idea of considering collections of objects rather than individual objects themselves was put into use. To this end, the corpse of bushes or grooves of trees or better still, jungles and forests were brought into use. The collection of all aquatic animals living in seas and oceans came to be called marine life. The sum total of all the raindrops was given the terminology of clouds. Mountains became the huge structures that are formed of stones and pebbles and may be countable if the size is pre-determined. It seems that although the world around us is not finite in its true form, but in order to make it more comprehensible and measurable, man has devised ways of categorizing it into discrete quantities, which give it the appearance of being finite.

It was the mathematician Georg Cantor (1845-1918) who in the early 1900's, laid the foundation of Set Theory and gave systematic definitions of countable and uncountable sets and in the process, discovered the hierarchy of infinities each larger than the preceding one (Mancosu, 2009; Dauben, 1990). This could be considered as some fantastic machinations of a super-imaginative mind. In fact, there was a lot of resistance to his theories by many of the renowned contemporary mathematicians such as Leopold Kronecker (1823-1891), Henri Poincare (1854-1912), Hermann Weyl (1885-1955), to name a few. Although Cantor's theories may belie intuition, but nature is replete with examples of countable and uncountable infinities. Let us understand the similarities between these and some mathematical examples.

Cantor gave the definition that any set of numbers that could be put in one-to-one correspondence with set of natural numbers ($\mathbb{N}$) is countably infinite. With this definition, the sets of integers ($\mathbb{Z}$) and rationals ($\mathbb{Q}$) are countable infinite sets, although they are apparently much larger than $\mathbb{N}$. This incongruity occurs when finiteness is broken into infinitely many parts. With this definition of countable infinity, one can easily see that all the trees, leaves and flowers are countably infinite, of course, taking into account that birth and death is a continuous process and is happening without any gap of time. The process of counting will be endless, but one can certainly begin the daunting task of counting these, starting from one corner of the earth and moving systematically around it, or else, different people can perform this task adopting their own areas. At each area, the number will be countably infinite. But then, infinity added countably many times will still result into countable infinity i.e.,

$$\infty + \infty = \infty$$

or more generally,

$$\infty + \infty + \cdots\cdots + \infty = \infty;$$

in fact,

$$\infty + \infty + \cdots\cdots = \infty$$

This takes care of things that can at least begin to be counted but if one wishes to count the number of raindrops in a water cloud or waves in an ocean or even stars in the galaxy? The problem is where to start and how do you ensure that in getting to the next raindrop or next star, you are not leaving out a few raindrops or a few stars in between. Similarly, it is not feasible to cut the waters in the ocean into countably many waves. These and many more are examples of another type of infinity present in nature which is not countable. There is a discreteness in countable elements. In other words, there are gaps between two successive elements in a countable set but there is continuity in an uncountable set i.e., there cannot be gaps in between any two elements. Since mathematics has been closely following nature in its travails, it had to produce a perfect example of this type of infinity in the form of real numbers (ℝ). There is no gap to be found between two real numbers. In fact, choosing a particular real number, it is not possible to list the next real number (Marwaha, 2016). With the discovery of real numbers, mathematics had completely transcended finiteness and it entered the domain of uncountable infinity where it encountered even more thrilling mysteries (Snow, 2003).

The life forms present in the entire universe can be regarded as examples of infinite quantities that are not countable. The 'Panspermia Hypothesis' supports this illusion (Kawaguchi, 2019). The Greek philosopher Anaxagoras (500 - 428 BC) propounded the theory of Panspermia (a Greek word, meaning 'seeds everywhere') which in modern times has taken the shape of a concept that believes in the seeds of life being scattered all over the universe and which can be propagated through space from one location to another. In the words of Anaxagoras:

'All things have existed from the beginning. But originally, they existed in infinitesimally small fragments of themselves, endless in number and inextricably combined. All things existed in this mass, but in a confused and indistinguishable form. There were the seeds (spermata) or miniatures of wheat and flesh and gold in the primitive mixtures, but these parts, like nature with their wholes, had to be eliminated from the complex mass before they could receive a definite name and character' (Marwaha, 2016).

3.1 Measuring The Infinite: Nature Inspires Set Theory

A set came to be known as a well-defined collection of objects; the expression 'well-defined' meaning a distinct property to be satisfied by the given collection of elements. Under this terminology, the tree may be regarded as a set of leaves, the properties of leaves defining the character of a tree, for example, a mango tree, a rose plant etc. The advantage of having the concept of a set is that a set can contain infinitely many elements as its members but the set itself can be regarded as a single entity. In mathematical terminology, a set is a collection of elements which satisfy some property $\mathcal{P}$.

In symbols, we denote a set as

$$S = \{x : \mathcal{P} \text{ is satisfied by each } x\}$$

With the help of this set-theoretic notation, collections which are impossible to list because of their infinite magnitude could easily be represented only by specifying the property satisfied by their elements. This is precisely what the mathematical genius Georg Cantor achieved after introducing the notion of sets.

If a set is finite and its elements can be listed, then its size is also a finite number. Cantor called the number of elements of a set as its cardinal number. Thus, a set having n objects had its cardinal number to be the natural number n. This way, all natural numbers $1,2,3,\cdots\cdots,n,\cdots\cdots$ represented finite cardinal numbers. But what about the set ℕ of natural numbers itself? Could it have a size? Since it was regarded as a set, it definitely looked like it had some size. So, Cantor accorded the size $\mathcal{N}_0$ (aleph-naught) to the countable infinity of natural numbers. He also decreed that sets which can be put in one-to-one correspondence with the set of natural numbers have cardinality $\mathcal{N}_0$. He went on to prove that this is true of integers and rational numbers and this amazing revelation had the sets of integers and rationals and even primes having the size $\mathcal{N}_0$.

$\mathcal{N}_0$ was the first infinite cardinal number. The next infinite cardinal number that mathematicians have discovered is 'c', the measurement of the size of real numbers. 'c' is also called the cardinality of the continuum, which is a nomenclature for the real number line.

Essentially what Cantor discovered was that not all infinite sets have the same size. There is a never-ending hierarchy of infinities, each larger than the preceding one. This was only the tip of the iceberg. The ramifications of Cantor's discoveries were wide-ranging, which led to intense and meaningful development of mathematics. On a more philosophical note, it also gave the message not to get overwhelmed by massive quantities or structures. By changing one's perspective of viewing them, it is possible to make them compliant and useful (Marwaha, 2016).

4. Nature Goes Mathematical

We have seen so far how nature inspired man to develop numbers and from there the journey of mathematics started. But what is amazing is to see these numbers being reflected in so many different aspects of nature (Goldsmith, 2012).

4.1 Fibonacci Numbers

Fibonacci numbers are attributed to the mathematician Leonardo of Pisa (1175-1240). They are also called the numbers of nature. Not only do they have special mathematical properties but are found in nature also (Goldsmith, 2012).The Fibonacci sequence of numbers is

$$1,1,2,3,5,8,13,21,34,55,89,144,\cdots\cdots$$

Note that each term of the sequence except the first two is the sum of its two predecessors.

Phyllotaxis is known as the study of geometry and structure of leaves, branches, flowers or seeds in plants. Studies have revealed that the plant kingdom has a curious preference for certain numbers and for certain spiral geometries and that these numbers and geometries are closely related (Vorobiev, 2002).

The number of petals in many flowers are Fibonacci numbers. For example, lilies and irises have 3 petals; parnassia and roses have 5 or 8; most daisies have 34, 55 or 89, marigolds have 13, sunflowers often have 55, 89 or 144 and chicory has 21 (Stewart, 2010).

We define another sequence formed of ratios of two successive numbers in the Fibonacci sequence.

$$\frac{1}{1},\frac{1}{2},\frac{2}{3},\frac{3}{5},\frac{5}{8},\frac{8}{13},\frac{13}{21},\frac{21}{34},\frac{34}{55},\frac{55}{89},\frac{89}{144},\cdots\cdots \qquad \text{(A)}$$

It is easily seen that the ratios gradually come closer and closer to the value 1.61803 (called the Golden Ratio or the Golden Number) and is denoted by Phi (φ) (Stewart, 2010).

$$\left(\frac{5}{3}=1.6666;\ \frac{8}{5}=1.6;\ \frac{13}{8}=1.625;\ \frac{21}{13}=1.615384;\ \frac{34}{21}=1.61904 \text{ and so on}\right)$$

Golden Ratio has been a subject of interest and interest for mathematicians of all ages starting from Pythagoras (580-500 BC) and Euclid (323-283 BC) in ancient Greece to Leonardo da Vinci (1452-1519) of Italy (Posamentier et al., 2011).

Here, we are not talking of employing mathematics to understand the natural phenomena but how certain aspects of nature behave mathematically. So, it is not incredible that mathematics became the language of nature. In fact, there is numerous evidence available that nature had been speaking this language much before man had learnt it. Nature propelled man to become aware of its presence in every activity taking place in natural surroundings and finding beautiful hidden patterns of geometry and numbers (Marwaha, 2016).

The Fibonacci fractions describe the growth of plants. When new leaves grow from the stem of a plant, they do not grow haphazardly, rather they form spirals while rotating in fractions of Fibonacci numbers. The turning of leaves takes place in a way that the new leaves do not block sunlight for the older ones and also that the rain water slides down to the roots easily. The same phenomenon exists between the petals of a sunflower. The number of spirals to the left is equal to the numbers of petals on the right and are generally adjacent numbers in the Fibonacci sequence (Grigas, 2013).

A square with the length of the side as a Fibonacci number is called a Fibonacci square. By adjoining the Fibonacci squares, we can form Fibonacci rectangles, or popularly known as Golden Rectangles. By joining the diagonally opposite vertices of the Fibonacci squares, a spiral is formed which is called the Golden Spiral.

Fibonacci ratios are believed to define beauty in the aesthetic sense, although nothing has been proved mathematically. The golden ratios and Golden Spirals are found in nature and also in different features of the human face. A face is considered to be symmetrically and classically beautiful if the features of the face are in Golden Ratio. Leonard da Vinci painted Monalisa using the Golden Rectangles and Golden Spiral. Although the Golden Ratio appears in art and architecture, even as far back as the Parthenon in ancient Greece or the Great Pyramid (Stewart, 2010), there are several illustrations of it in nature. Golden Spiral is seen in hurricanes and tornadoes. Even the shape of the egg forms the Golden Spiral. The seeds on the heads of sunflowers and daisies arrange themselves in spiral patterns (Stewart, 2010); the petals of rose flowers and seashells are some more examples of Golden Spiral in nature (Vorobiev, 2002; Grigas, 2013).

4.2 The Number Pi

No article in mathematics can be considered complete without the mention of the number pi (denoted by symbol π). If, amongst the family of numbers, we start looking for the most charismatic and intriguing number, the crown will unanimously be put on the head of ‘pi’. It is an irrational number whose decimal representation is without an end and without a pattern, in a layman’s language. The value of pi to 20 decimal places is

$$\pi = 3.14159265358979323846\cdots\cdots$$

The first calculation of π is attributed to Archimedes but what makes it amazing is that it is linked to nature in a most beautiful and fascinating way.

Professor Han-Henrik StØllum, an earth scientist at Cambridge University, wanted to calculate the sinuosity or bendiness of the rivers worldwide, that is, the ratio between the actual length of rivers from source to mouth and their direct length as the crow flies (Singh, 2005). Professor StØlum published in his paper “River meandering as a self-organization process” (StØllum, 1996) that although the ratio varies from river to river, the sinuosity ranges between 2.7 and 3.5 and the average of all the numbers resulted in pi.

Citing StØllum’s findings, Singh writes (Singh, 2005, pp. 17):

“The number π was originally derived from the geometry of circles and yet it reappears over and over again in a variety of scientific circumstances. In the case of the river ratio, the appearance of π is the result of a battle between order and chaos. Einstein was the first to suggest that rivers have a tendency towards an ever more loopy path because the slightest curve will lead to faster currents on the outer side, which will in turn result in more erosion and a sharper bend. The sharper the bend, the faster the currents on the outer edge, the more the erosion, the more the river will twist and so on. However, there is a natural process which will curtail the chaos; increasing loopiness will result in rivers doubling back on themselves and effectively short-circuiting. The river will become straighter and the loop will be left to one side forming a lake. The balance between the two opposing factors leads to an average ratio of π between the actual length and the direct distance between source and mouth. The ratio of π is most commonly found for rivers flowing across very gently sloping plains, such as those found in Brazil or the Siberian tundra."

5. Nature, Geometry And Topology

Mathematics owes its existence to nature and its innate characteristic is to derive inspiration from natural elements. Since nature functions methodically and logically according to its laws, the evolution of Mathematics to become the language of nature has been logical and rational. It perceives reality and aims to comprehend it in a very organised and consistent manner. For example, prior to understanding a 3-dimensional figure, say, a rectangular box, first a series of logical definitions will have to be made. The box is made up of six plane surfaces joined in a certain manner. Unless the plane surface is properly defined, it is not possible to comprehend the structure of the box. Next, the plane is composed of lines, which are made up of points. The mathematical point cannot be broken down further and so the development of the field of geometry begins with the definition of a point, line, plane and so on.

This could not be a figment of one's imagination. Now a circle, for example, is a perfect figure where every point on it is equidistant from the centre. There are circular shapes present in nature. The contour of the sun had been a perfect model for a circle till the time it was discovered to be a spherical ball of gases and fire. But the man has also seen the moon change its contour every night, although the continuity in the change is not visible. The different phases of the moon have set the imagination rolling defining the arcs and sectors of a circle.

At the formative stage, it was nature that inspired the human brain to think of geometrical shapes and other aspects related to them like their areas, circumference, perimeter etc. At an advanced stage of the subject's development, this and many other examples could have paved the path for the journey from geometry to topology.

Topology is a discipline which allows geometrical figures to be continuously stretched, twisted or bent without cutting or tearing them. This can be compared to a surface which is made from a flexible dough (to be regarded as a thin sheet and not a solid lump) that can be deformed and twisted to another shape but cannot be broken or torn into two pieces. For this reason, topology is often referred to as 'rubber-sheet' geometry. In the nineteenth century, mathematicians proved that any two-sided surface without a boundary is topologically equivalent to a sphere, a torus (doughnut), a torus with two holes, a torus with three holes, and so on (Stewart, 2010). Although the subject of topology is only a century or so old, nature had been giving indications of it since the very beginning. The clouds assume all sorts of shapes and sizes with the change being noticeably gradual and not abrupt. Another example is that of the Mobius band which has connections with the behavioural pattern of mind (Marwaha, 2016).

5.1 Mind, Nature And The Mobius Band

The Mobius band or Mobius strip is a topological surface with only one side. The credit of discovering it goes to the German mathematician August Ferdinand Mobius (1790 – 1868) in the 19th century. It is obtained by giving a half twist to a rectangular strip and joining its two ends. Unlike the rectangular paper, where a bug placed on one side of it will have to go over the edge to get to the other side, if the same bug were to crawl along the length of this strip, it would return to its starting point having traversed the entire length of the strip without crossing the boundary. Thus the Mobius band has only one side and the edge of a Mobius strip is topologically equivalent to a circle (Stewart, 2010).

The transition from geometry to topology bears a testimony to the fact that mathematics is indeed a true spokesperson of nature. Simple examples of 2-sided surfaces in nature are leaves. They provide a natural resting abode for bugs and other small creatures. Dew or raindrops settle on the cringes of the leaves to quench the thirst of these hapless insects. A leaf or a twig can get naturally twisted and become a one-sided surface to the utter delight of a bug that can easily slide all along the leaf without having to change its orientation.

The topology behind creating a mobius strip converts a 2-dimensional surface into a 3-dimensional entity (Mobius band). Mind which is a repository of both negative and positive thoughts can be regarded as a Mobius band-like entity. The innate characteristic of the mind is to be constantly in a state of contemplation and it transmits effortlessly from positive to negative thinking and vice-versa. As long as there is a twist in the thought process, the mind will remain enmeshed in this habit pattern of vacillating between wholesome and unwholesome emotions. Moreover, it can lose its ability (orientation) to disseminate between the right and the wrong. In order to have a balanced mind, it is imperative for it to possess razor-edged discriminatory ability to help it to overcome its vicissitude.

5.2 Chaos Theory and Fractals

In the eighteenth century AD, the genius of Newton gave the laws that described and determined the behaviour of natural and cosmic elements. Newton's laws were quite simplistic for they predicted the future of nature based on the conditions prevalent in the present day and time. But this is not the complete truth about nature. Its behaviour is stochastic and can never be fully determined. No single formula can encompass the vastness of the universe. The molten lava erupting from volcanoes and earthquakes gives us a fair estimate of what lies at the core of the earth but the changes occurring there from moment to moment cannot be gauged. one tiny change that appears to be slight at initial stages can trigger off a mammoth reaction beyond anyone's imagination. This is an instance of disorderliness or chaos present in the natural systems that is exceedingly difficult to fathom (Stewart, 2010; Marwaha, 2016).

The proponent of Chaos Theory was the mathematician and meteorologist Edward Lorenz (1917 – 2008). In his calculations of the weather on the computer with an inbuilt system of processing information, he found that an insignificant change in the initial numbers, which was not even an error, fed in the computer produced entirely different results. This came to be known as the 'butterfly effect', signifying that a slight change perpetuated by the flap of a butterfly's wings can bring about a totally random and chaotic effect in the atmosphere (Stewart, 2010). It was observed that if these occurrences happen often enough, they also come in the predictable zone. But again, this is not the final answer. Then, is there a way out of this conundrum? The mathematicians working on the Chaos Theory strive to find simple equations that will give solutions with not too many aberrations, despite the change in the initial conditions. This has also led to the study of fractals propounded by mathematicians Benoit Mandelbrot (1924 - 2010) and Helge Von Koch (1870 -1924).

Fractal Geometry is one of the mathematical fields whose origins can be found in a natural phenomenon known as The Coastline Paradox. In the year 1950, the English mathematician Lewis Fry Richardson (1881-1953) wanted to measure the length of the border shared between the countries of Portugal and Spain. When he looked up the official length, he noted that the two countries reported two different values for their shared border length and what was surprising was that these values differed by more than 200 kilometres. How could two measurements for one and the same object be completely different? This was the starting point of a new mathematical entity known as 'fractals'. Fractals are geometrical entities which generate self-similar patterns that are fractions but occur in a regular manner (Marwaha, 2016).

In 1967, Benoit Mandelbrot, a French mathematician explained that it was impossible to obtain an exact measurement of the coastline because of its irregular rugged terrain with rocks jutting out to the sea. Satellite pictures will gauge a shorter length as the coastline will appear to be very smooth from that distance. A person walking along the coastline will take into account all the juttings and indentations and therefore the length will be considerably larger for him. On the other hand, an insect traversing the shoreline would give a very high estimate because it cannot ignore even the tiny pebbles or grains of sand. Likewise, for a microbiologist who would want to measure it at the atomic or molecular level, the length will be off the charts. Thus different scales of measurement will yield different lengths (Marwaha, 2016).

Mandelbrot invented a new field of mathematics, 'fractal geometry', that studies the fractal dimension of such irregular curves and surfaces whose lengths cannot be measured. He discovered new geometrical structures suitable for describing these irregular sets of points, curves and surfaces from the natural world. He coined the word "fractals" for these entities. There are several examples of fractals in the human body, formation of clouds, trees, mountain surfaces etc (Stewart, 2010). As applications of fractals in nature, scientists are now able to forecast natural disasters such as hurricanes, floods, earthquakes and volcanic eruptions using fractals (Devlin, 1988).

6. Paradoxes

Paradoxes are an intrinsic part of nature. In our day-to-day life, we come across so many inconsistent, puzzling and absurd instances of behaviour of nature. Water, the most basic ingredient for existence of life, is distributed in a very erratic manner. While desert areas cry out for water, the tsunamis and hurricanes deluge and destroy life. But still oases exist in deserts and fish thrive in excessive water. Mountains have originated from the seabeds and water bodies like lakes and rivers exist on the mountains. Dinosaurs with developed organ systems could not survive through the Ice Ages whereas tiny creatures like cockroaches with a simple body plan have lived through all the hazardous environmental changes and perhaps is the only species which will outlive all life forms including human beings in the event of an apocalypse. Nature is bursting with many more such paradoxical instances. But have these examples thwarted the evolution process in any way? In fact, an incisive look into all this leads to a better understanding of how nature wants us to adapt and evolve (Marwaha, 2016).

The same phenomenon is applicable to mathematics. The journey of mathematics has been rife with paradoxes. It all started during the ancient Greek civilisation in the fifth century BC with Zeno of Elea, a Greek philosopher who lived around 460 BC, propounding many paradoxes. This was the time when mathematics was still in a very formative stage of development. Mathematical infinity had not been discovered and calculations were restricted to adding only finite quantities and that too only finitely many times. This led to many paradoxes. Since we are exploring the mathematics-nature connection, we shall only focus on those paradoxes that can be explained through examples occurring in nature. We illustrate two of Zeno's paradoxes which are familiar and relatable to real life situations and do not involve any complex mathematics (Huggett, 2018).

Zeno explained that although in the fable of the race between the hare and the tortoise, the tortoise won the race as the hare chose to sleep on the way but mathematically, it can be proved that even if the hare had not dozed off, there was no way he could have overtaken the tortoise. The reason for this was simply that a quantity that can be broken into an infinite sum of positive numbers cannot be finite; in this case, the quantity being time (or distance). This is a paradox as it seems quite improbable that a fast paced hare can be defeated by a snail paced tortoise in a race.

Suppose the tortoise is given a head start by the hare (because of his cockiness) and $t_1$is the initial time taken by the hare to catch up with him but within this time, the tortoise would have moved ahead. Again, let t_2 be the time taken by the hare to cover this much distance during which the tortoise has crawled further. According to Zeno, no matter how fast the hare is, he will never be able to catch up with the tortoise because the time taken by him will form an infinite series

$$t_1 + t_2 + t_3 + \cdots$$

which should have an infinite sum.

For the same reason, a man trying to reach to the other end of a room will never be able to do so because the total length of the room say, l, can be broken into an infinite sum of finite lengths. This is obtained by observing that the man will first traverse half of the length of the room ($\frac{l}{2}$), then half of the remaining length ($\frac{l}{2^2}$) followed by half of the remaining length ($\frac{l}{2^3}$) and so on. At this rate, there will always be some distance of the room which will remain untraversed and the total length covered will be the infinite series

$$\frac{l}{2} + \frac{l}{2^2} + \frac{l}{2^3} + \cdots = l\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right)$$

Zeno's period was the time in history when limits and limiting processes of infinite sums of infinitesimal quantities had not been discovered and properly defined. In all practicality, the sum of infinitely many numbers should be infinite only. With the invention of infinitesimal Calculus, by two contemporary giants, viz., Isaac Newton (1642-1727) and Gottfried Wilhelm Leibniz (1646-1716), Zeno's paradoxes were solved (Bell, 2013). It was understood and mathematically explained that in the limiting process, quantities that are becoming smaller and smaller, add up to a finite sum. Since it was proved that

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 1,$$

the total distance covered by the man trying to cross a room of length l is l and therefore it is possible for the man to cross the room from one end to the other. Similarly, as the time gap between Achilles and the tortoise is reducing indefinitely (mathematically speaking, $t_n \to 0$), Achilles will eventually take over the tortoise and win the race.

7. Conclusion

The journey of mathematics continued its roller coaster side with ups and downs, high and lows but always emerging a winner with nature constantly by its side as a guide and a true friend. The more man sought the proximity of nature, the more he discovered beautiful hidden patterns that led to the discovery of geometry, topology (rubber-sheet geometry), projective geometry and more recently the fuzzy structure of fractals which are surfaces having fractional dimensions (Devlin, 1988). It would be only proper to mention here that evolution of mathematics in even the more advanced areas as computers had historic origins. Ancient Egyptians knew that all numbers can be expressed as a power of two and modern computer programmers also know and use the same idea.

This article will not be complete without the mention of the Greek mathematician Pythagoras (580 – 500 BC) who coined the word 'philosopher'. It was Pythagoras's love for mathematics that fostered his belief in the philosophy of life. As and when he discovered the amazing beauty of numbers, he was compelled to reflect upon the spiritual aspect of nature. Pythagoras believed that all the secrets of the universe could be explained through numbers. For him, the numbers held the key to the mysteries of nature. He founded a school of mathematicians (The Pythagorean Brotherhood) who revelled and exulted in discovering amazing properties of numbers. Each new discovery strengthened their belief that the numbers were a concrete realisation of the abstract phenomena of nature and its elements. One of such discoveries was the relation between the music and numbers. He observed that for a stringed instrument to generate harmonious notes, the strings must be fixed at points which bear a simple ratio to the length of the string, i.e. the points should have a ratio of exactly one-third, one-fourth, one- fifth, etc. otherwise, the notes produced will be disharmonious. He attributed the following characteristics to the numbers one to ten: Number 1: The number of reason; Number 2: The number of diversity; Number 3: The number of harmony; Number 4: The number of retribution; Number 5: The number of marriage; Number 6: The number of creation; Number 10: The number of the universe (Marwaha, 2016).

I would like to conclude with the following observation. Apart from the commonalities between mathematics and nature, there is a deep connection of these two with the mental faculties of humans. Mind, mathematics and nature are intertwined and tend to influence one another in both abstract and tangible manner.

Learning mathematics is like learning a technique through which the phenomena that govern the tangible and the non-tangible world can be understood. It is perhaps the only bridge linking reality to abstraction. Mathematics teaches to be self-reliant and broad-minded. It is a training to think logically and methodically and arrive at the right conclusion. The basis for any discovery lies at the power of intuitive thinking and deduction based on observations. But once intuition has paved the way for certain conclusions to follow, the validity of these conclusions must be established. The strength of mathematics lies in validating what it believes in with proof. 'To prove is to believe' defines the character of mathematics. Common intuition led to the discovery of numbers (Law, 2012).

It was only a sense of strong intuition that led the Greek mathematician Euclid (325 – 265 BC) of Alexandria, known as the "father of geometry' to postulate (Euclid's Fifth Postulate) that 'through any point not on a given line, one and only one line can be drawn parallel to the given line' (Ravindran, 2007). For a sceptic, who cannot accept anything without a sound logical explanation, it is very difficult to digest this parallel postulate as an unalterable truth (Smarandache, 2000). Many mathematicians tried to find a proof for it but failed. It was independent of all other postulates, which in simple words mean that it is possible to construct a self-consistent system of geometrical statements by deductions from a set of axioms in which the parallel postulate is replaced by a contrary postulate. This was eventually done and it ultimately led to the formation of a system called non-Euclidean geometry. But it did not make the Euclidean geometry vulnerable or faulty in any way. Projective geometry or elliptic geometry came to be known as Riemannian geometry as the idea was propounded by the mathematician Riemann (1826-1866). The essence of the idea is that a line is parallel to another line as long as it can be seen, but how does one verify the parallelism between two lines until infinity. One can only be certain of the unboundedness of the lines but cannot confirm that they will not meet at infinity. So, where Euclid's Parallel Postulate was based on the faith that what appears to be true in front of eyes will remain true away from sight, Riemann and other mathematicians believed that two lines may intersect at infinity. This brought about a revolution in the way of thinking of scientists and the non-Euclidean geometry led to many important discoveries, the Theory of Relativity by Einstein (1879-1955) being one of them. To sum it up, nature does not want us to be bound to any belief; it wants us to experiment in different ways and it is fine as long as every new path leads to non-contradictory truthful conclusions (Marwaha, 2016, pp. 184-185).

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