Spirituality: Through the Lens of Mathematics

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***Abstract**: This paper tries to establish the deep connection of Mathematics to Spirituality by exploring the characteristics of beauty, truth and selflessness that are embodiments of God or Nature. Beauty is reflected in the complex patterns of Nature and mathematicians have come closer to God through developing mathematics as the language of nature. Truth resides in both physical and metaphysical manifestations of nature and the journey on the path of spirituality is completed when both these aspects are comprehended. Mathematics serves as a bridge between these two absolute truths. It unravels the mysteries of the universe to embark on the quest for Truth or God and also to dig deep into the mind to recognize and perceive the subconscious levels which actually control and guide our existence. In this paper, we explore all the different affiliations of mathematics to spirituality.*

Keywords: God, Nature, Truth, Beauty, Benevolence, Sixth Sense, Observation, Representation, Proof, Yin and Yang, Platonism, Ego, Cantor Set

1. Introduction

According to nineteenth century English mathematician Hilda Phoebe Hudson (1881-1965), "to all of us who hold the Christian belief that God is true is a fact about God, and mathematics is a branch of Theology" (Hudson, 1925).

The key to spirituality is the freedom of the spirit. Desires give rise to negativities, which tend to bind the spirit. It loses its urge to soar high and far seeking unknown lands of peace, truth and happiness. Mathematics is encompassed with an aura of spirituality because it possesses an unbridled spirit. It is not afraid to explore the deep, hidden principles of nature, and after having discovered these, it expresses them in a most logical manner without trepidation or hesitation. It will not accept anything short of truth and absolute truth.

To be spiritual is equivalent to training the mind to become the master of the present moment; free of negativities. The nature of the subject of mathematics is such that it does not allow the mind to indulge in idle thinking which tends to generate defilements of ego, animosity, craving, greed, fear etc. As long as the mind is immersed in doing mathematics, clouds of self-doubt and fear of the future are dispelled and this gives the mind the freedom to discover the fundamental truths of life. In a way, mathematics helps to bring the student closer to God because one feels like a creator, and the unbridled joy of being constructive is an experience of Godhood (Meiyappan, 2017).

We all have our notion of spirituality. For me, it means understanding the vagaries of one's mind; learning to discipline it and purifying it by eradicating its propensities to generate negativity and eventually filling it with positive and wholesome thoughts which should ultimately manifest into positive actions, beneficial for one and all. When the mind struggles to overcome tendencies to generate anger, jealousy, ego, hatred, greed, ill will, animosity, impatience and a range of other impure emotions, it looks for a viable solution and this is where mathematics comes to the rescue. Mathematics is not illusionary; it is deeply attached to the universe. It is the tool through which the illusory world can be made tangible. Mathematics can be called the 'mind of the universe' as it has evolved to give a precise meaning to the universe and its laws (Marwaha, 2016).

The omnipotent powers of mathematics to unravel the mysteries of the universe with 'unreasonable effectiveness', a phrase coined by the Nobel Laureate Eugene Wigner (1902 – 1995), made him wonder whether God is a mathematician (Livio, 2009). The three forms of God, each complete in themselves, are truth, beauty and compassion. Realising any one of these three in entirety results in realising God himself. The conception of mathematics is an honest attempt to cover all these three aspects of spirituality. Galileo Galilei (1564-1642) said, "Mathematics is the language in which God has written the universe".

God can also be considered a disciplinarian because He is bound to function within the laws of nature. Anything which conforms to the rules and regulations has to be visually beautiful and spiritually appealing. The infinite beauty of God is reflected in nature. Beauty is not that which appeals only to the senses. The British mathematician Godfrey Harold Hardy (1877-1947) quotes in his book *"A Mathematician's Apology"* (Hardy, 1940),

"The mathematician's patterns, like the painter's or the poet's, must be beautiful, the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test; there is no permanent place in the world for ugly mathematics. It may be very hard to define mathematical beauty, but that is just as true of beauty of any kind – we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognising one when we read it."

Ancient Greeks and Romans understood the beauty of proportionality and used the golden ratio to create visually and aesthetically appealing masterpieces of art and architecture. The Greek temple Parthenon in Athens was constructed around the fifth century BC using the golden proportion (1 : 1.618). The irrational number with approximate value 1.618 is called the golden number and is generally denoted by the Greek letter φ (phi). The saga of golden ratio continued through the centuries and got a new meaning when in the fifteenth century Leonardo da Vinci (1452 – 1519) depicted the golden proportion in his paintings of the human body. The enchanting face of Monalisa bears testimony to this rule (Devlin, 1988; Vorobiev, 2002). The celebrated Welsh mathematician and philosopher Bertrand Russell (1872-1970) was of the opinion that "Mathematics rightly viewed possesses not only truth but supreme beauty".

Mathematics influenced human life in many ways. Art in any form is a physical manifestation of all that is pure, noble and spiritual in the human mind that is seeking expression. Mathematics has more than fulfilled its part in making this expression a thing of sublime beauty. It is only through mathematics that art of painting saw a major revolution during the Renaissance period. Using mathematical techniques of projection and perspective, flat surfaces could give the illusion of three-dimensional space, making them look life-sized and realistic (Courant et al., 1996).

Truth and mathematics are synonymous entities. Both cannot exist without one another. The laws of nature needed the language of mathematics to bring to the fore their existence and significance, which completely adheres to the principles of truth. This language not only experiences the truth but interprets it and draws its conclusions. The beauty of mathematics is that it is the embodiment of truth. It aspires after the truth and the path it chooses is truth itself. Last but not least, mathematics inadvertently makes one appreciate all aspects of nature, especially its benevolence and magnanimity (Marwaha, 2016). The German astronomer and mathematician Johannes Kepler (1571-1630) said

"The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which he revealed to us in the language of mathematics"

Mathematics is a subject that is so disciplined that it does not give itself allowances to behave in an arbitrary manner. Just like nature, it is bound by laws and axioms, violation of which is not permissible. The medium through which these laws are expressed is called the language of mathematics. Similarly, nature illustrates its fundamental laws through natural principles at the external level, and through mind-matter phenomena at the spiritual level (Kramer, 1995).

2. Sixth Sense: The Power Of Intuitive Thinking

Sixth sense has been gifted to man by nature to perceive things which are not so obvious and which the five senses fail to comprehend. Sixth sense can be called the power of intuition or the ability to transcend the mundane and worldly to reach out to the ultimate, unknowing realm of truth. Thus, this sixth sense is not controlled by the mind. This works perfunctorily at its own rate and on its own terms. It resides in the vast untapped reservoir of the unknown part of the mind which has the power to feel and cognize the truth more intensely and scrupulously. The psychologists draw comparisons between the mind and an iceberg through the Iceberg Model, to explain the two levels of consciousness of the mind, one superficial and the other superconscious. The surface level of the mind which is responsible for cognition is like a tip of the iceberg, a mere four or five percent of the total mind. The remaining part of the mind, known as subconscious or unconscious, constitutes the rest of the gigantic part of the iceberg which remains submerged in water (Green, 2019).

Mathematics is actually the progeny of the sixth sense of the human brain. It is an outcome of the endeavour of man, at the subconscious level to grasp the real meaning of our existence. It is a manifestation at the physical level, of the spiritual arena of the human mind. The yearning to realise the ultimate truth; the frustrations at the inability to do so; the flashes of ingenuity; the fleeting moments of triumph turning into despair; and the stage of irrevocable comprehension of truth; are the aspects which can be said to aptly describe the spiritual and mathematical journey of mankind.

Intuition and instinct have formed an intrinsic part of mathematical thinking. Although the ultimate truth is accepted after a rigorous proof is given, intuition has always been the guiding spirit that purports logical and rational explanation of facts. Perhaps the greatest example of powers of supernatural being at work in area of mathematics is that of Srinivasa Iyengar Ramanujan (1887-1920), a young, self-taught mathematical prodigy from a poor family in South India who floored the elite mathematical fraternity of Cambridge University with his simple and intuitive brilliance to do mathematics (Kanigel, 1991). The Scottish mathematician and science fiction writer Eric Temple Bell (1883-1960) has hailed Ramanujan as a gift from heaven with supernatural insight into apparently unrelated formulas (Bell, 1936).

There is a famous story of Ramanujan who was a wizard with numbers. Once when terminally ill, he was visited in the hospital by G. H. Hardy. As an opening remark of the conversation, Hardy mentioned that the cab in which he had travelled had a very dull number viz. 1729 in the mathematical sense, to which Ramanujan immediately replied that Hardy was grossly wrong. Ramanujan said that it was the smallest number that could be expressed as the sum of two cubes in two different ways, that is,

$1729 = 9^3 + 10^3 = 1^3 + 12^3$. Even in the face of death, Ramanujan had not lost his ability to think rationally. This instance speaks volumes about the sheer power of mathematics to help keep the mind of a person in full control.

Endurance is the greatest virtue as it is a synonym of dispassionate observation. When we desist from confrontation, storms arise and pass away without inflicting any harm. A continued practice of this attitude brings forth the need for accepting the laws of nature and adapting to them. These traits, ‘endurance’ and ‘adaptation’, accrue from the sixth sense, which is also known as the ‘basic instinct’ or the ‘survival instinct’. Unless this instinct is developed and utilized, there will be no progress towards understanding the laws, which govern our very existence. Intuition is the honest call of the conscience from the depths where only truth prevails.

Mathematics, as we know it today, would not have existed, if man had not cared to listen to his voice of conscience. And, the conscience echoes the directives of nature. If man had not keenly observed the innumerability of objects present in nature, he would not have had the perspicacity to accept the denumerability of natural numbers. It was only a strong sense of intuitive thinking that led Euclid (325 - 265 BC) to postulate the Parallel Axiom that 'through any point not on a given line, one and only one line can be drawn parallel to the given line' (Ravindran, 2007).

The remarkable feature of this axiom is that it makes an assertion about the whole extent of a straight line, that is, it says that two lines which are parallel do not intersect, no matter how far they may be produced. Now, visually this axiom can only be verified for a finite distance and not for the entire length line extending up to infinity (Courant et al., 1996). Here is an axiom, which transcends finiteness and enters the realm of infinity. It is easy for an intuitionist to believe in the axioms of Euclidean Geometry but for a formalist, who cannot accept anything without a sound logical explanation, it is very difficult to digest this parallel postulate as an unalterable truth. Many mathematicians tried to search for a proof of Euclid's Parallel Postulate but they all failed. An unending record of failures in finding the proof led to the belief that perhaps the postulate was independent of the rest of the axioms and thus different systems of geometry could be constructed in a self-consistent system in which the parallel postulate is replaced by a contrary postulate. This was eventually done and it ultimately led to the formation of a system called a non-Euclidean geometry (Meyer, 2006). The revolutionary importance of the discovery of non-Euclidean geometry lay in the fact that it demolished the notion that the only mathematical framework into which physical reality can be fitted must be based on the Euclid's axioms (Marwaha, 2022). This example establishes the credibility of intuitive thinking that springs from unflinching faith in the unknowability and uncertainty of regions which can be reached only through honestly listening to the calling of conscience that resides there.

3. Observation, Representation and Proof

"In this life, there are three kinds of men, just as there are three sorts of people who come to buy and sell, the next above are those who compete. Best of all, however, are those who come simply to look on. The greatest purification of all is, therefore, disinterested science, and it is the man who devotes himself to observation is able to deliver himself from the wheel of birth" - Bertrand Russell

The first step on the path of spirituality begins with the observation of truth as it is, without trying to change it in any way. It is human nature to interfere with everything due to the egotistical and self-righteous attitude of the mind and then, on the basis of its past experiences, the mind develops the habit of evaluating a situation as good or bad. This proves to be a great impediment on the path of spirituality. Here lies another connection of mathematics to spirituality because the approach of mathematics to grow and evolve is through observation and analysis of the facts, followed by a mathematical representation and lastly prove them in all truthfulness. True to its innateness to be spiritual, mathematics functions at the following three levels: -

3.1 Observation

Observation entails a will to know the reality. Mathematics provides a meticulous training of the mind to become a dispassionate yet objective observer of the natural phenomena. The journey of mathematics begins with the observation of the evidence pointing towards the existence of numbers, whole or fractional, countable or uncountable. The process of addition and multiplication was happening all around. Nature had a way of bringing together entities with similar properties which led to the concepts of sets and groups (Kramer, 1966).

To illustrate the power of keen observation, we give the example of the role of numbers in creating musical harmony (Schopenhauer, 1966). Iamblichus (245 CE – 325 CE), a fourth century Greek scholar, researched and wrote extensively on Pythagoras (570 – 490 BC) and his Brotherhood. According to him, Pythagoras happened to pass one day by a blacksmith's shop and heard the sounds produced by hammers striking the iron. The perspicacious powers of Pythagoras made him observe that certain hammers whose masses were simple ratios of each other produced harmonious sounds and the hammers whose weights had no relationship with others generated disharmonious sounds. This discovery led Pythagoras to claim that "all nature consists of harmony, arising out of numbers" (Taylor, 1818).

3.2 Representation

Observation must be followed by the right expression. For a comprehensive understanding of the working of nature, a proper language was needed. Thus the language comprising symbols, their meanings and functions was developed. The set theory was a logical representation of the way subjects were handled in day-to-day life. Farmer A gifted a basket of apples and farmer B gifted a basket of oranges to farmer C on his birthday. Farmer C's wife stored them together in a heap, which was nothing but the union of the two baskets. Similarly, when the farmer's wife singled out mangoes from different baskets containing heterogeneous fruits, she was inadvertently performing the mathematical operation of intersection between two or more sets. These are extremely simple illustrations but these are present all around us in abundance. Only a keen mental perception is needed to realise their significance and utilize it to develop a subject. This subject has been mathematics, wherein all the basic axioms and fundamental principles are inspired by such examples present in nature around us (Peat, 1990).

David Hilbert (1862 – 1943) was a stalwart German mathematician who lent an aura of clarity, precision and sophistication to mathematics. He proposed a set of twenty-three unsolved problems at the International Congress of Mathematicians in Paris in 1900 in which he put into perspective many mathematical themes that he thought could set the course for further research. It was only due to a proper representation of these problems that incisive thinking into these areas led to a proper understanding and significant development of modern mathematics (especially in Set Theory, Axiomatic Theory and Logic). Hilbert gave a touch of elegance to the way mathematical problems must be posed, represented and proved. He belonged to the class of mathematicians who believed in representing mathematical ideas in the most logical, precise and axiomatic form so that these not only reflect the accuracy and rationale of mathematics but also prove the consistency and completeness of the subject. His dream of proving that an axiomatic approach of mathematics is consistent and complete did not materialize into reality, nevertheless Hilbert remains the face of the present day mathematics (Singh, 2005).

3.3 Proof

It is not such a difficult task to formulate a language based on observation, but what lends credibility to it is that it should represent truth and nothing but the truth and that is the hallmark of spirituality for any discipline. Mathematics has followed this approach of becoming a spokesperson of nature and truth right from its inception. This is evident from the fact that no statement can be issued without being supported by a proof. This proof must be in conformation with the infallible and unchallengeable laws that form the building blocks of mathematics. In the seventeenth century, the French mathematician Pierre de Fermat (1607-1665) posed a seemingly simple problem, "Does the equation $x^n + y^n = z^n$, $n \geq 3$ have any solutions in positive integers?". This was a natural generalisation of a similar question for the equation $x^2 + y^2 = z^2$, the solution of which was given by Pythagoras in the sixth century BC. After Pierre de Fermat's death, a note was found scribbled in the margin of a page that the solution to the general problem was in the negative and that he had found a perfectly marvellous solution but was unable to jot it down due to lack of space. Since Fermat was known to be a highly prolific mathematician, few had any reason to doubt the veracity of his words. Thus for three and a half centuries, mathematicians strove hard to solve the problem and some succeeded to obtain only partial solutions. But the absolute proof eluded the mathematicians until in 1993, Andrew Wiles (b.1953), a professor in Princeton University, accomplished the impossible by solving the problem after labouring over it for more than seven years (Singh, 2005).

One of the crucial links in the proof of Fermat's Last Theorem by Andrew Wiles was provided by a conjecture posed by Japanese mathematicians Yutaka Taniyama (1927-1958) and Goro Shimura (1930-2019) in 1955. The conjecture involved the relation between two areas of mathematics, viz. modular forms and elliptic equations which are not even remotely related to one another. At the time of announcement of this conjecture, it was claimed to be highly bizarre and a result of apparent wilful thinking. Although, it was a thought that was to bring a revolution in the field of number theory, nevertheless it had no claims to its veracity without a proof. Taniyama had shown that the relation between these two unrelated areas verified for some cases that he had tested but it was still a conjecture only without a proof that works for all cases (Marwaha, 2016). The equivalence of the truthfulness between Fermat's Last Theorem and what came to be known as Taniyama–Shimura Conjecture shifted the focus from proving Fermat's Last Theorem to proving this conjecture. Even establishing the equivalence between the two results was a major breakthrough but the real problem remained unsolved.

Mathematics can play around and come up with wonderful and aesthetically beautiful connections between seemingly unrelated areas but up to the time, the veracity of these links is established beyond doubt, these will remain to be clever machinations of contriving minds. Only a veritable proof can lend credibility and sensibility to an otherwise beautiful but soulless piece of art. This is only one instance among several which highlights the significance of 'proof' in mathematics. Eventually the Taniyama –Shimura Conjecture was proved by Andrew Wiles and this proved what Fermat had claimed to be true (minus the proof), three and a half centuries ago (Singh, 2005).

4. Yin and Yang – Opposites are a Way of Life

Life is a mix of opposites. Every event that occurs, whether in the physical world or in the mental and emotional world, has two faces to it; one is apparent and the other hidden. The ancient Chinese philosophers observed the fundamental principle that the characteristic of opposites is present in every aspect of nature. They called these opposite entities – Yin and Yang. Yin, generally regarded as female, represents the negative principle in nature and Yang, the male counterpart, is representative of the positive principle. Here positive or negative aspects are not to be judged as good or bad but to be regarded as relative terms. For example, if Yin is the moon, then Yang is the sun. Both moon and the sun have their respective qualities which cannot be permanently described as beneficial or harmful. Yin-Yang are reflected in the different seasons, day and night, water and stream, matter and energy. In all these examples, one important fact that comes to light is that Yin-Yang are not in a stationary state; in fact, they are constantly changing forms from one into the other, at the same time being in a state of equilibrium (Guoli et al., 2021).

"What a caterpillar calls the end of life, wise men call a butterfly" - A Chinese proverb.

The Yin-Yang model can also be used to understand not only the physical but the mental and the spiritual nature of human dialectical thinking. The mind is always replete with thoughts. Some thoughts are infused with positivity while others are negative and harmful. The driving force behind these thoughts is the ego which has the power to convert negativity into positivity and vice-versa. The ego is blind and for it, the supreme truth is the self and its nature. Therefore, it reacts with closed eyes according to its prejudices, opinions, impressions and conditionings. This constantly affects the balance of Yin-Yang in the mind and the spiritual strength of a person lies in discerning both the facets with clarity and precision. But before doing this, the Yin-Yang factor should be deeply impressed upon the mind that nature is a blend of extremes (Marwaha, 2016).

Mathematics is a true child of nature, as the essential quality of having pairs of opposites is amply exhibited in its spirit. According to Zhang and Shao (Zhang et al., 2012), in Chinese philosophy, all the rules established by human beings can only be done in sync with its understanding and development of nature. They further elucidate that according to traditional Chinese mathematics, a mathematical object can be studied only in a model-free structure by researching through the complex relationships in non-compatible and paradoxical conditions. The following examples are given to illustrate this line of thought.

Example 1. The advent of counting numbers was not enough to sustain the credibility of mathematics. These numbers accounted for finite quantities but in the real world, the feeling of nothingness and deficit is omnipresent. Thus, it was imperative that the number zero and negative natural numbers be included. To every natural number p, there corresponds a negative natural number $-p$. These are represented on the opposite sides of the number line.

Example 2. In defining many of the basic concepts of mathematics, the definition can be approached through two directions. For defining the limit of a function, we first define its limit inferior and limit superior which always exist and if these happen to be equal, then their common value is defined to be the limit of the function. The value of the limit lies between the limit inferior and limit superior.

Example 3. With the definition of the Riemann Integral of a function are attached the two concepts of upper integral and the lower integral and only when these are equal do, we say that the Riemann Integral of the function exists; otherwise, the value of the integral lies between the lower and the upper integrals. One interesting fact to note here is that the upper and the lower integrals always exist, though they may not be equal.

Example 4. Every function has a positive part and a negative part and it is expressible as their difference. If 'f'denotes a function and f^+, f^- respectively its positive and negative parts, then

$$f = f^+ - f^-, \quad \text{where}$$

$$f^+(x) = \max\{f(x), 0\} \text{ and } f^-(x) = \max\{-f(x), 0\}$$

Mathematics is replete with such examples where pairs of extremes come together to lend credibility to a concept. In following a particular line of thought, extreme situations can occur that threaten to upset the balance of rational thinking. But whatever the outcome of this upheaval might be, it is never at the cost of truth. The spirit of mathematics has always remained untainted and victorious whenever there has been a war of conflicting and extreme schools of thought (Marwaha, 2016).

4. Ego and The Cantor Set

The journey on the path of spirituality begins with self-introspection and ends with self-realisation. This is the essence of Platonism, the philosophy of the ancient Greek philosopher Plato (427-347 BC), that all spiritual experiences are transcendental and independent of their actual existence in the physical world. Kessler speaks about this in his paper (Kessler, 2019) in relation to spirituality in mathematics. Many eminent mathematicians such as Gottlob Frege (1848-1925), Kurt Friedrich Gödel (1906-1978) and Bertrand Russell have endorsed this viewpoint.

Notwithstanding the objections and counter arguments given by many philosophers to mathematical Platonism, I would like to substantiate it with the example of Cantor Set, a mathematical entity that can be exemplified as a physical representation of the abstract vagaries of the human mind. The analogy of the Cantor Set elucidates a very significant aspect of path to spirituality and that is overcoming the egotistical attitude that prevents from looking at the reality as it is (Marwaha, 2016).

Let us begin with understanding the process of construction of Cantor Set, which in itself is quite revelatory in nature. The mathematician Georg Cantor (1845-1918) started with the closed unit interval [0, 1]. We all know that the length of this interval is 1. Divide the interval into three equal parts and remove the open middle third i.e., the open interval $\left(\frac{1}{3},\frac{2}{3}\right)$ from$[0,1]$. What remain are two closed intervals $\left[0,\frac{1}{3}\right]$ and $\left[\frac{2}{3},1\right]$ of length $\frac{1}{3}$ each. Again, divide these two intervals into three equal parts and remove their respective open middle thirds. This will be $\left(\frac{1}{9},\frac{2}{9}\right)$ and $\left(\frac{7}{9},\frac{8}{9}\right)$, each having length $\frac{1}{3^2}$. After this second phase of elimination, four closed intervals viz. $\left[0,\frac{1}{9}\right]$,$\left[\frac{2}{9},\frac{1}{3}\right]$, $\left[\frac{2}{3},\frac{7}{9}\right]$ and of length $\frac{1}{9}$ or $\frac{1}{3^2}$ each is remaining. Now, if we divide each of these closed intervals into three equal parts and remove their respective middle third, we shall be left with eight closed intervals of length $\frac{1}{3^2}$ each. Let us continue indefinitely this process of eliminating parts of [0, 1]. At the nth stage, 2^{n-1} intervals of length $\frac{1}{3^n}$ each is removed. Therefore, in the limiting stage, the total length of the intervals removed will be

$$\frac{1}{3}+2\cdot\frac{1}{3^2},+2^2\cdot\frac{1}{3^3}+2^3\cdot\frac{1}{3^4}+\cdots\cdots+2^{n-1}\cdot\frac{1}{3^n}+\cdots\cdots$$

$$=\frac{1}{3}\left(1+\frac{2}{3}+\frac{2^2}{3^2}+\frac{2^3}{3^3}+\cdots\cdots\right)$$

$$=\frac{1}{3}\cdot\frac{1}{1-\frac{2}{3}}$$

$$=1$$

If from an interval of length 1, chunks and chunks of points are taken away which all add up to length 1, then what do you expect to remain in the interval? On the surface level, it appears that [0, 1] has been totally emptied of all points. It was the genius of Cantor that he discovered that despite this deletion, [0, 1] still has infinitely many points left in it. Moreover, these infinitely many points are uncountable in number. Isn't this a mind-boggling revelation? The interval [0, 1] has uncountable infinity of points to begin with and if from this interval of length 1, a part of unit length is removed, still there are infinitely many points remaining. The collection of these infinitely many points is called the Cantor Set. (Some of the points which are clearly visible in this set are the end-points of the open intervals viz., $\frac{1}{3},\frac{2}{3},\frac{1}{9},\frac{2}{9},\frac{7}{9},\frac{8}{9},$ etc. which do not get removed at any stage) (Dauben, 1990).

The Cantor Set is one of those examples of mathematics where its claim to spirituality is substantiated. Let us compare the mind to the Cantor Set. The mind is a storehouse of thoughts and emotions. If driven by ego, hatred and anger, we throw venom on others thinking that it will help to clear our mind of negativities, then we are grossly mistaken. It makes no dent in our existing stock of impurities. In fact, by generating negativities, we shall only succeed in multiplying the impurities, which will spell further doom for us.

5. Conclusion

In this paper, we have seen several illustrations to show that the mathematical concepts are closely associated with the phenomena of nature on one hand and the creator, on the other. This line of thought is in sync with many mathematicians, philosophers and scientists from Plato to Stephen William Hawking (1942-2018). In his book “Plato and Aristotle in Mathematics”, Anglin writes that “Plato believes that in studying mathematical truths, we shall be in a better position to know the absolute, necessary truths about what is good and right and thus be in a better position to become good ourselves” (Anglin et al., 1995).

According to Stephen Hawking, “I am not religious in the normal sense. I believe the universe is governed by the laws of science. The laws may have been decreed by God, but God does not intervene to break the laws.” The real beauty of mathematics lies in the fact that it embraces different perspectives and philosophies. For this reason alone, mathematics can be claimed to be spiritual because the essence of spirituality lies in giving freedom of thought and respecting everyone’s understanding of the ultimate truth.

I would like to conclude by bringing to light the debate that has been going on among the mathematicians ‘whether mathematics is discovered or invented?’. This question is closely related to the perception that an individual might have regarding the spiritual connection of mathematics with nature.

Quoting Albert Einstein (1879-1955) – “How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?”, this statement of Einstein highlights the dichotomy that exists in the nature of mathematics as a discipline, and its perception by an individual. On one hand, there is mathematical Platonism where the eternal mathematical truths exist intrinsically in nature and it is up to us to discover them. “The knowledge of which Geometry aims is the knowledge of the eternal” – Plato. On the other hand, there is this non-Platonist way of thinking that believes that reality is far more complex and to comprehend it fully, elegant methods have to be invented, which also need to be experimentally verified (Abbott, 2013).

The irrational number Pi (π) is an incomprehensible irrational number with no familiar pattern extending endlessly towards eternity, but strangely has connections with nature (Singh, 2005), raising the quandary if π is contrived or exists naturally? Similarly, complex numbers appear naturally in many areas of physics and engineering, but for an engineer, there is nothing magical about complex numbers, even though the Euler’s remarkable formula $e^{-i\pi} = -1,$ which seems so far off from reality helps to describe a rotation by π radians (Hamming, 1980).

Platonists view fractals as proof of a mathematical entity that has an independent existence in nature. But the connection of fractals to nature might not be visible to a mathematician who is lost in iterative formulae that are calculative and contrived products of the human mind (Penrose, 2004).

To conclude the paper, I would like to say that whether mathematics is invented or discovered or whether it is spiritual or steeped in atheism, it definitely provides the bridge for the human beings to contemplate on the questions of our existence and its connection with nature. It paves the path for contemplation and inspires to find answers and even if the solutions are elusive and hard to find, every step taken on the path is meaningful and worthwhile. Albert Einstein has beautifully summed up the dichotomy in approach and perception of mathematics in his following quote: “As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality”.

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