Approximate Reasoning in Linear Programming

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Abstract: In this article, we would like to attempt to find an almost optimal solution to a class of linear programming problems (LPP) in which the objective as well as the constraints are so imprecisely defined that it is practically impossible to represent them by any existing crisp model. The aim is to propose a method to solve such an imprecisely defined problem without directly transforming it to a crisp one at the structural level so as to provide more flexibility to the decision-maker. To solve such a problem we use well known fuzzification, aggregation and defuzzification techniques as available in approximate reasoning methodology. It is concerned with the design of an iterative algorithm for a convergent near optimal satisfactory solution starting with an initial solution from a not-so-clear feasible region. Artificial examples have been considered to illustrate the proposal and existing results are used for comparison. A comparative study revealed interesting and satisfactory results.

Keywords: Fuzzy Linear Programming Problem (FLPP), approximate reasoning, fuzzification, aggregation, defuzzification

1. Introduction

Linear programming constitutes a part of an important area of Applied Mathematics called 'constrained optimization techniques' where, the agenda is to optimize an objective under constraints. The classical problem aims to find the minimum or maximum of a linear function under constraints which are represented by linear (in)equations. However, in a typical problem, often the decision-maker might not really want to optimize (maximize or minimize) the objective function, rather (s)he might want to attain some satisfaction (aspiration) level which might not even be definable in precise term. For example, the decision maker might wish to **improve the present sales** situation considerably and thereby, generate **substantial profit** for his organization.

Fuzzy linear programming problem (FLPP) was introduced to capture such form of imprecision in linear programming problems (LPP). The concept of maximizing decision under uncertainty was proposed by Bellman and Zadeh [4]. Later, various modification methods appeared from different interpretations and the knowledge of such a semantic representation, thereby, becomes more meaningful and extensively applicable.

Most proposals, as of today, for fuzzy optimization are based on Bellman-Zadeh's concept and on transformation of the problem to a crisp one [17]. In [15], the authors computed solution of multi objective fuzzy linear programming problem and showed that solutions are independent of weights. Zimmermann in [3] and [18] presented fuzzy approaches to linear

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programming problems. Fuzzy linear programming problem with fuzzy coefficients was formulated by Negoita [13]. Dubois and Prade investigated optimization with linear fuzzy constraints [7]. An advanced method to solve LPP is presented in [16], where decision variables, cost coefficients involved in the objective function and right hand side coefficients in the constraints are represented with trapezoidal fuzzy numbers. Using multiplication and addition operations of trapezoidal fuzzy numbers together with linear ranking function, the FLPP is converted to a crisp LPP and eventually solved by well-known simplex method. In [14], a wide variety of methods are considered for solving Fully Fuzzy Linear Programming Problem(FFLPP). An efficient method was introduced in [6] to solve FFLPP which is derived from the multi-objective LPP and lexicographic ordering method. There, the authors considered a theoretical analysis for their proposed method. In [5], the authors developed an efficient algorithm for solving FFLPP. It was a combination of Charnes-Cooper scheme and the multi-objective LPP. Furthermore, application of the proposed method in real life problems was presented and compared with some existing methods. In the sequel, Angelov proposed a fuzzy optimization technique based on logical aggregation and defizzification operator [1]. Based on a generalization of Bellman-Zadeh's concept, a second attempt is made for fuzzy optimization in [2]. The two methods designed by Angelov are non-iterative algorithms for solving fuzzy linear programming problems. It has been observed that for different goals these two methods actually lead to significantly different solutions.

In all these researches, either the imprecise problem was first converted to a precise LPP and then solved by simplex algorithm to obtain an optimal solution of the allied LPP or they were based on the generation of a decisive set obtained from some combination of the constraints with the objective followed by defuzzification, wherever necessary.

In this paper, we propose an iterative approach which provides a convergent satisfactory solution for different goals starting with an initial solution from the not-so-tight feasible region of the problem at hand. Our proposal consists in finding a solution to a FLPP which is a three-stage process based on fuzzification, logical aggregation and defuzzification. We have observed that the proposed iterative method actually leads to a converging solution. This is the novelty of the proposal — whatever be the interpretations of the fuzzification, aggregation and/or defuzzification, even the simplest of all can produce almost the same result as that obtained using the most sophisticated one present in the existing literature.

The paper is organized into five sections. After a brief introductory Section 1, we discuss, in Section 2, the mathematical formulation of our proposed iterative technique to solve fuzzy linear programming problem based on the well known existing aggregation - defuzzification process as proposed by Bellman and Zadeh. In Section 3, we develop an algorithm for finding a solution of a FLPP. It is illustrated with an artificial example. In a Sub-section, there is a comparative study of different existing techniques for solving FLPP based on Bellman and Zadeh's aggregation - defuzzification operators with our proposed method. The paper is concluded in Section 4, followed by a list of references in the last section.

2. Mathematical Formulation

In representing human understanding of real world activities, Zadeh in 1965, introduced for the first time, the concept of a fuzzy set. A fuzzy set F over the universe of discourse U is conveniently represented by the collection of ordered pairs of the form

$$F = \{(u, \mu_F(u)) | u \in U, \mu_F(u) > 0\} \text{ where } \mu_F : U \rightarrow [0,1].$$

Usually, fuzzy sets are used to model linguistic expressions like tall, short etc., which describe imprecise value of an attribute (, say height) of an object. Thus, we may interpret the characteristic function for an ordinary set to be generalized to a membership function that assigns to every element of U a value from the unit interval $[0,1]$ instead of $\{0,1\}$. μ_F is used to denote the degree of membership of u in F which associates with each u in U, a real number in [0,1].

In the year 1970, Bellman and Zadeh [4] proposed a general structure for tackling fuzzy decision making that can be summarized as

$$D = G \cap C$$

where D is to denote the fuzzy decision from the composition of G, the fuzzy goal and C, a set of fuzzy constraints. If x is to denote a set of alternatives/decisions then a fuzzy decision making model can be interpreted as one of finding x by the aggregation of the goal G and the constraints C.

This approach gives the decision set to be a fuzzy set resulting from the combination of the fuzzy sets of satisfaction level corresponding to the goal and the constraints. In this framework, Zimmermann [17] formulated FLPP as in the following:

$$\widetilde{\max}\ Cx$$
$$\text{such that } A\,x \lessapprox b$$

where, A, b, C are parameters of the mathematical form and are given by $A \in R^{m\times n}$ (the activity parameter), $b \in R^{m}$ (the requirement parameter), $C \in R^{n}$ (the cost coefficients), $x \in R^{n}$ is a vector of decision variables. Here, $\widetilde{\max}$ is used to denote fuzzy maximization and $\lessapprox$ is a fuzzy inequality.

A solution of this problem is a fuzzy set, called the decision fuzzy set $D(x_i)$, $i = 1,2,\dots,n$ for each decision variable x_i obtained from aggregated decision fuzzy relation $B(x_1, x_2, \dots, x_n)$. First of all, we discretize the domain of definition of $x_1, x_2, \dots, x_n$ and let us call them as $U_{x_1}, U_{x_2}, \dots, U_{x_n}$. We then fuzzify the objective and the constraints of the programme. Then we construct the decision fuzzy relation on the universe of discourse $U_{x_1} \times U_{x_2} \times \dots \times U_{x_n}$. This constitute the aggregation. Here, we aggregate membership degree of objective as well as constraints by min conjunction operator. This aggregation may be performed in many other ways because of the flexibility of different fuzzy logics. Here, $\mu_0(x_i)$ is to represent the membership degree of the element x_i with which the objective of the problem is being satisfied. Whereas, $\mu_j(x_i)$ is the degree of membership of x_i, in the satisfaction level of the jth - constraint of the problem $j = 1,2,\dots,m$. All the arguments i.e., $x = (x_1, x_2, \dots, x_n)'$ can be approximated from the feasible region of the given problem. The output generation begins with projection of the relation individually on the respective universe of discourse of the decision variables of the problem. Each such projection results in the formation of a fuzzy set over $U_{x_i}; i = 1,2,\dots,n$.

A crisp solution, wherever necessary, is obtained by defuzzification of the decision fuzzy set by any suitable method like the one given below:

$$\overline{x_i} = \frac{\sum_{j=1}^{i_k} x_{ij}\mu_D(x_{ij})}{\sum_{j=1}^{i_k} \mu_D(x_{ij})}$$

where, $\mu_D(x_{ij})$ is the membership of the corresponding x_{ij} from decision fuzzy set $D(x_i)$. Now, any such x satisfying the constraints of the FLPP is a feasible solution of the problem. A fuzzy LPP is said to be consistent if it admits of a feasible solution. In this paper, we propose to design an iterative scheme for a convergent feasible solution of any consistent FLPP.

This four stage fuzzification, aggregation, projection and defuzzification process will be continued unless value of $C\bar{x}$ will approximately be equal in two consecutive iterations.

3. Algorithm – illustration

Input: A fuzzy LPP in standard mathematical form:

$$J(x) = \widetilde{\max}\ Cx$$
$$\text{such that } A\,x \lessapprox b$$

Output: A converging solution corresponding to an initial feasible solution and the associated decision.

Step 1: (Initialization) Set a goal arbitrarily and denote it by J_* (Determine the same from experience, expectation from knowledge or the estimation obtained from a group of skilled persons).

Step 2: (Fuzzification) Based on a choice of J_* we set the membership functions for satisfaction of the uncertain objective i.e., μ_0 as well as the constraints μ_j, $j = 1,2,\dots,m$ (these are the satisfaction level of objective of the programme together with all the constraints of the programme; all expressed as fuzzy sets)

$$\mu_0 = \begin{cases} 1; & if\ J(x) > J_* \\ \frac{J(x)-J_0}{J_*-J_0}; & if\ J_0 \leq J(x) \leq J_* \\ 0; & if\ J(x) < J_0 \end{cases}$$

$$\mu_j = \begin{cases} 1; & if\ \sum_{i=1}^{n} a_{ji}x_i < b_j \\ 1 - \frac{\sum_{i=1}^{n} a_{ji}x_i - b_j}{\delta_j}; & if\ b_j \leq \sum_{i=1}^{n} a_{ji}x_i \leq b_j + \delta_j \\ 0; & if\ \sum_{i=1}^{n} a_{ji}x_i > b_j + \delta_j \end{cases}$$

where $j = 1, 2, \dots, m$.

Here, δ_j is the tolerance or threshold value for constraints.

Step 3: (Discretization) Fix some feasible region with not-so-tight boundary (any region about a solution of the associated crisp problem that can generate the objective of the programme satisfying the constraints) for the given problem and discretize the same to obtain: $U_{x_1} = \{x_{11}, x_{12}, \dots, x_{1l_1}\}$ i.e., l_1 no. of points, $U_{x_2} = \{x_{21}, x_{22}, \dots, x_{2l_2}\}$ i.e., l_2 no. of points,…, $U_{x_n} = \{x_{n1}, x_{n2}, \dots, x_{nl_n}\}$ i.e., l_n no.
of points, where each U_{x_i} denotes the domain of the decision variable $x_i, i = 1,2,\dots,n$.

Step 4: (Decision relation) For each n−tuple, combine the membership values of satisfaction of the fuzzy set corresponding to the objective with all the fuzzy sets representing the constraints by choosing a suitable conjunction operator (e.g., the 'minimum' conjunction operator). Thus, we obtain the aggregated decision fuzzy relation $B(x)$ i.e., $B(x_1, x_2, \dots, x_n)$ over $U_{x_1} \times U_{x_2} \times \dots \times U_{x_n}$ as in the following:

$$B(x_1, x_2, \dots, x_n) = \min\{\mu_0(x_{1j_1}, x_{2j_2}, \dots, x_{nj_n}), \mu_1(x_{1j_1}, x_{2j_2}, \dots, x_{nj_n}), \dots, \mu_m(x_{1j_1}, x_{2j_2}, \dots, x_{nj_n})\},$$

where $1 \leq j_1 \leq l_1$, $1 \leq j_2 \leq l_2$,…,$1 \leq j_n \leq l_n$.

There are exactly $l_1 \times l_2 \times \dots \times l_n$ elements each with having a certain degree for being a feasible solution of the given programme.

Step 5: (Projection) Let us project the aggregated decision fuzzy relation $B(x)$ over the appropriate domain successively to obtain decision fuzzy set $D(x_i)$, $i = 1,2,\dots,n$ corresponding to each decision variable x_i as in the following:

$$D(x_i) = Proj_{U'(x_i)}^{B_i(x_1, x_2, \dots, x_n)}, \quad U'(x_i) = U_{x_1} \times U_{x_2} \times \dots \times U_{x_n} \text{ (except } U_{x_i}\text{)}$$

$= \sum_j sup\, \mu_{B_i}(x_{ij})/x_{ij};\quad 1 \le j \le l_k$,

where $B_i(x_1, x_2, \dots, x_n)$ denote degrees of satisfaction of arguments of U_{x_i} (containing l_k no. of points) in $B(x_1, x_2, \dots, x_n)$.

Thus, we obtain

$D(x_i) = \mu_D(x_{i1})/x_{i1} + \mu_D(x_{i2})/x_{i2} + \dots + \mu_D(x_{il_k})/x_{il_k}$,

here each membership is chosen by maximum membership value corresponding to each argument.

Step 6: (Defuzzification) Let us choose an appropriate defuzzification method, say,

$$\overline{x_i} = \frac{\sum_{j=1}^{l_k} x_{ij}\mu_D(x_{ij})}{\sum_{j=1}^{l_k} \mu_D(x_{ij})}$$

for each $i = 1,2, \dots, n$ to defuzzify each decision fuzzy set of the problem, where Σ denotes the algebraic summation, x_i is the i^{th} component of the decision vector x and $\mu_D(x_{ij})$ is the corresponding membership.

This, in turn, generates some value $J(x)$ for the objective function.

Step 7: (Repetition) If the solution is not upto a desired degree of accuracy then we set this value of J as the new objective J_* for the next iteration and set μ_0 again depending on new J_*. Go to Step 4. Otherwise, we stop and conclude that the value of J is the solution that we can achieve with prescribed satisfaction level and x is the corresponding decision.

To illustrate the same let us consider a fuzzy linear programming problem in which all the relations appearing in the constraint are fuzzy.

Example 1:

$$J = \widetilde{max}\ 20x_1 + 25x_2$$

$$\text{such that} \quad 6x_1 + 10\,x_2 \precsim 660$$

$$0.5\,x_1 + 0.3\,x_2 \precsim 47.$$

First we fuzzify the above problem. For this we consider the value of J_* i.e., goal as 2200. The membership functions for objective as well as the constraints are set as in the following:

$$\mu_0 = \begin{cases} 1; & if\ J > 2200 \\ \dfrac{20x_1 + 25x_2 - 2000}{200}; & if\ 2000 \le J \le 2200 \\ 0; & if\ J < 2000 \end{cases}$$

$$\mu_1 = \begin{cases} 1; & if\ 6x_1 + 10\,x_2 < 660 \\ 1 - \dfrac{6x_1 + 10\,x_2 - 660}{40}; & if\ 660 \le 6x_1 + 10\,x_2 \le 700 \\ 0; & if\ 6x_1 + 10\,x_2 > 700 \end{cases}$$

$$\mu_2 = \begin{cases} 1; & if\ 0.5\,x_1 + 0.3\,x_2 < 47 \\ 1 - \dfrac{0.5\,x_1 + 0.3\,x_2 - 47}{3}; & if\ 47 \le 0.5\,x_1 + 0.3\,x_2 \le 50 \\ 0; & if\ 0.5\,x_1 + 0.3\,x_2 > 50 \end{cases}$$

Let us take the core of the feasible region of the given problem at hand. Accordingly, the following discrete values of arguments are considered as feasible for further processing to achieve the desired goal. Let them be given by

$x_1 = \{80, 85, 90, 95\}$ and $x_2 = \{10, 15, 20, 25\}$.

Now, aggregated fuzzy relation i.e., for each pair of the above arguments the satisfaction will be as computed and presented in Table 1:

Table 1. Decision fuzzy relation

	80	85	90	95
10	0	0	0.25	0
15	0	0.375	0.17	0
20	0.5	0	0	0
25	0	0	0	0

Now, decision fuzzy set corresponding to x_1 and x_2 are as follows:

$D(x_1) = 0.5/80 + 0.375/85 + 0.25/90$

$D(x_2) = 0.25/10 + 0.375/15 + 0.5/20$.

Now, let us consider the defuzzified values as $\overline{x_i} = \frac{\sum_{j=1}^{4} x_{ij}\mu_D(x_{ij})}{\sum_{j=1}^{4} \mu_D(x_{ij})}, i = 1,2$.

Hence we obtain,

$$\overline{x_1} = \frac{80 \times 0.5 + 85 \times 0.375 + 90 \times 0.25}{0.5 + 0.375 + 0.25} = 83.89,$$

$$\overline{x_2} = \frac{10 \times 0.25 + 15 \times 0.375 + 20 \times 0.5}{0.25 + 0.375 + 0.5} = 16.11, \qquad i.e., J = 2080.55$$

For convergence of solution, we continue to apply this technique repeatedly taking the value of J obtained in this stage as J_* for use in the next iteration. Accordingly, the result of iterations is tabulated as in the following with the decision variables in the objective function as well as in the constraints being taken as fuzzy numbers. We observed that this convergent solution (J=2073) is close to the actual solution (J=2075) of the associated defuzzified problem. A finer discretization may produce a more close value of J.

Table 2. Solution using our proposed technique

Aggregation Operator	*Defuzzification Operator*	*Iteration*	J_*	*Solution*		
				x_1	x_2	J
min	*weight average method*	1*st*	2200	83.89	16.11	2080.55
		2*nd*	2080	85.30	14.70	2073.50
		3*rd*	2073	85.39	14.61	2073.05

3.1 Comparative Study

Here, we would like to compare our result with that of Angelov's fuzzy optimization techniques [1, 2]. It has been observed that solutions obtained by Angelov's technique differ significantly from that obtained by proposed method. We also solve the same problem using some other existing aggregation [2] - defuzzification [8] operators. It has been found that the solution obtained by proposed technique and that obtained by existing techniques are almost close to each other for some problems whereas they differ significantly for most others. We are able to show that our convergent solution technique is independent of choice of operators and application results are satisfactory.

Problem 1: $J = \widetilde{max}\ 20x_1 + 25x_2$

such that $6x_1 + 10\ x_2 \lesssim 660$

$0.5\ x_1 + 0.3\ x_2 \lesssim 47.$

Problem 2: $J = \widetilde{max}\ 2x_1 + 3x_2$

such that $x_1 + 2\ x_2 \lesssim 4$

$3\ x_1 + x_2 \lesssim 6.$

Table 3. A comparison of solutions with Angelov's technique

Problem	*Angelov's Method*	*Proposed Method*
Problem 1	x_1 =87.1 x_2 =15.2 J=2122.0	x_1 = 85.39 x_2 =14.61 J=2073.05 *No. of iterations*: 3
Problem 2	x_1 =2.08 x_2 =1.8375 J=9.6725	x_1 =1.421 x_2 =1.46 J=7.222 *No. of iterations*: 2

Table 4. Solution using different aggregation and defuzzification operators

Problem	*Aggregation Operator*	*Defuzzification Operator*	*Solution: J*
Problem 1	min	*weight average method*	J=2073.05
		mean of maximum	J=2075.00
	algebraic product	*weight average method*	J=2075.65
		mean of maximum	J=2075.00
	$\max(\alpha+\beta-1, 0)$	*weight average method*	J=2076.50
		mean of maximum	J=2075.00

Table 3 shows a comparative study between Angelov's methods [1, 2] and our proposed technique for two fuzzy linear programming problems. The first problem was considered by Angelov and the second one was considered by Gasimov's in [10], Let us mention here that for the second problem, we found solutions 5.76 and 6.73 by considering Intuitionistic fuzzy approaches as given in [11, 12] whereas, the crisp solution of the associated defuzzified problem is 6.8. In case of Angelov's problem, the achieved optimal J value 2122 can also be obtained following our technique. However, since the solution is not found to be convergent at that stage of iteration, we have to continue the process and thereby obtain a J value close to 2075. An important drawback in Angelov's proposal is that, there is no general characterization on the choice of γ, δ *and* α which are present in aggregation and defuzzification operators. Moreover, in Angelov's technique, arguments of decision variables are considered arbitrarily but in our technique, these are taken from the feasible region of the corresponding defuzzified problem. The result of application of different existing aggregation and defuzzification operators to our proposed technique are shown in Table 4, where, we see the solution lies in between 2073 and 2076 due to different choice of operators. We also observed that the solution obtained from our iterative method is pretty close to solution for the associated defuzzified Angelov's problem which is 2075 and that for Gasimov's problem [10] is 6.8 .

In this case, if we choose J_* as greater than its previous value then obviously the solution increases. In our technique, there is a flexibility in the choice of the goal. If in the initial iteration, J_* is less than the value of J then, we can achieve the same J iteratively. Further, if the goal is considered larger than the initial one then, we can also get the same solution, only difference is that the number of iterations may be greater than that in the previous case which is shown in the Table 5.

Table 5: Flexibility in choice of the goal

Problem	*Iteration*	*Goal* : J_*	*Solution* : *J*
Problem 1	1st	**2050**	J = 2073.65
	2nd	2073	**J = 2073.05**
	1st	**2100**	J = 2075
	2nd	2075	J = 2073.03
	3rd	2073	**J = 2073.05**
	1st	**2200**	J = 2080.55
	2nd	2080	J = 2073.5
	3rd	2073	**J = 2073.05**
	1st	**2400**	J = 2089.55
	2nd	2089	J = 2074.20
	3rd	2074	J = 2073.05
	4th	2073	**J = 2073.05**
	1st	**2600**	J=2100.75
	2nd	2100	J=2075
	3rd	2075	J=2073.05
	4th	2073	**J=2073.05**
	1st	**3000**	J=2114.70
	2nd	2114	J=2075.94
	3rd	2075	J=2073.05
	4th	2073	**J=2073.05**
Problem 2	1st	**5.5**	J = 7
	2nd	7	**J = 7.22**
	1st	**7**	**J = 7.22**
	1st	**8**	J = 7.44
	2nd	7	**J = 7.22**
	1st	**12**	J = 7.55
	2nd	7	**J = 7.22**
	3rd		
	1st	**20**	J=7.78
	2nd	7	**J=7.22**
	3rd		
	1st	**35**	J=7.78
	2nd	7	**J=7.22**

4. Conclusion

We have shown that a standard FLPP, where the coefficients associated with the decision variables in the objective function as well as in the constraints are taken as fuzzy numbers, can be solved using approximate reasoning methododlogy. The proposed iterative method is a flexible one as we can choose the operations in a number of ways, leading to different values in the output. The difference with other existing techniques is that the algorithm actually leads to a converging solution — whatever be the choice of the initial solution and/or interpretation of the fuzzification, aggregation and defuzzification procedures. Here, we choose discrete values of arguments from the coarse feasible region of our proposed problem and aggregate them by min operation, a well-known simple conjunction operator. Generation of decision fuzzy relation constitutes the core of this research work. Projection help us in making a fuzzy decision on a fuzzy problem. Defuzzification may be done following any one of the existing techniques. Use of weight average method yields satisfactory result. We conclude that tools and techniques of approximate reasoning methodology may be conveniently used in fuzzy optimization.

There is considerable scope for further research in this domain. This includes, in particular, effect of variation in l_0 and δ_j to our solution along with application of the proposed algorithm to different models of FLPP.

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